Spin-charge rotating local reference frames: a unified U(2)=U(1)⊗SU(2) approach to the interacting electrons

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A spin-charge unifying description for the Hubbard model based on the time dependent local gauge transformations is developed. The collective variables for charge and spin are isolated in the form of the space-time fluctuating U(1) phase field and rotating spin quantization axis governed by the SU(2) symmetry, respectively. As a result interacting electrons appear as a composite objects consisting of bare fermions with attached U(1) and SU(2) gauge fields. We elaborate on the microscopic origins of the effective action with the Coulomb interaction that contain topological theta terms. Furthermore, we unravel the link between nontrivial multiply-connected topological structure of the U(2)=U(1)⊗SU(2) configurational space for gauge fields and the instanton contribution to the statistical sum.

1 Introduction

Among the electronic Hamiltonians relevant for interacting systems the Hubbard model [1, 2, 3] is considered as one that contains the essential ingredients for understanding the physics of correlated electrons. In a many-body system the relevant physics is encoded in the symmetries and in the Hubbard model they are represented by the charge U(1) gauge and spin rotational SU(2) groups relevant for the occurrence of the superconducting and magnetic orderings. Interestingly, the resulting U(2)=U(1)⊗SU(2) symmetry group has a nontrivial multiply connected topological structure. For multiply connected configuration spaces novel features can arise as for example in the Aharonov-Böhm effect [4] governed by the multiply connected U(1) group manifold. Since the homotopy class π_1[U(2)]$= Z$ also forms a set of integer winding numbers the topological structure of the configuration space is nontrivial, ambiguities may arise when attempts are made to specify a value for the phase of a wave function for the whole configuration space [5]. Thus the problem we are facing is that of many-body quantum mechanics on a multiply connected configuration space. In the present work we develop a spin-charge unifying description for interacting electrons given by the Hubbard model. It is based on the time dependent local U(2) gauge transformations to disentangle the Coulomb interaction. The collective variables for charge and spin are isolated in a form of the space-time fluctuating U(1) phase field and the rotating spin quantization axis governed by the SU(2) symmetry, respectively. As a result interacting electrons appear as composite objects consisting of bare fermions with attached U(1) and SU(2) gauge fields and due nontrivial topology of the U(2) group an instanton contribution to the statistical sum emerges.

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2 Hubbard Hamiltonian

Our starting point is the purely fermionic Hubbard Hamiltonian $\mathcal{H} \equiv \mathcal{H}_t + \mathcal{H}_U$:

$$\mathcal{H} = -t \sum_{\langle \mathbf{r}\mathbf{r}' \rangle, \alpha} [c^{\dagger}_{\alpha}(\mathbf{r}) c_{\alpha}(\mathbf{r}') + \text{h.c.}] + U \sum_{\mathbf{r}} n_{\uparrow}(\mathbf{r}) n_{\downarrow}(\mathbf{r}). \tag{1}$$

Here, $\langle \mathbf{r}, \mathbf{r}' \rangle$ runs over the nearest-neighbor (n.n.) sites, t is the hopping amplitude, U stands for the Coulomb repulsion, while the operator $c^{\dagger}_{\alpha}(\mathbf{r})$ creates an electron with spin $\alpha = \uparrow, \downarrow$ at the lattice site $\mathbf{r}$, where $n_{\alpha}(\mathbf{r}) = c^{\dagger}_{\alpha}(\mathbf{r}) c_{\alpha}(\mathbf{r})$. Usually, working in the grand canonical ensemble a term is added to $\mathcal{H}$ in Eq.(1)to control the average number of electrons, $\mathcal{H} \rightarrow \mathcal{H} - \mu \sum_{\mathbf{r}} n(\mathbf{r})$ with μ being the chemical potential and $n(\mathbf{r}) = n_{\uparrow}(\mathbf{r}) + n_{\downarrow}(\mathbf{r})$ the number operator. It is customary to introduce Grassmann fields, $c_{\alpha}(\mathbf{r}\tau)$ depending on the "imaginary time" $0 \leq \tau \leq \beta \equiv 1/k_B T$, (with T being the temperature) that satisfy the anti–periodic condition $c_{\alpha}(\mathbf{r}\tau) = -c_{\alpha}(\mathbf{r}\tau + \beta)$, to write the path integral for the statistical sum $\mathcal{Z} = \int [\mathcal{D}\bar{c}\mathcal{D}c]\, e^{-\mathcal{S}[\bar{c},c]}$ with the fermionic action

$$\mathcal{S}[\bar{c}, c] = \mathcal{S}_B[\bar{c}, c] + \int_0^{\beta} d\tau \mathcal{H}[\bar{c}, c], \tag{2}$$

that contains the fermionic Berry term [6]: $\mathcal{S}_B[\bar{c}, c] = \sum_{\mathbf{r}\alpha} \int_0^{\beta} d\tau \bar{c}_{\alpha}(\mathbf{r}\tau) \partial_{\tau} c_{\alpha}(\mathbf{r}\tau)$ that will play an important role in our considerations.

3 Spin-charge U(2) reference frames

For strongly correlated system it is crucial to construct a covariant formulation of the theory which naturally preserves the spin-rotational symmetry present in the Hubbard Hamiltonian. In order to maintain spin rotational invariance, one should consider the spin-quantization axis to be a priori arbitrary and integrate over all possible directions in the partition function. For this purpose the density–density product in Eq.(1) we write, following Ref.[7], in a spin-rotational invariant way:

$$\mathcal{H}_U = U \sum_{\mathbf{r}} \left\{ \frac{1}{4} n^2(\mathbf{r}\tau) - [\mathbf{\Omega}(\mathbf{r}\tau) \cdot \mathbf{S}(\mathbf{r}\tau)]^2 \right\}, \tag{3}$$

where $S^a(\mathbf{r}\tau) = \frac{1}{2} \sum_{\alpha\alpha'} c^{\dagger}_{\alpha}(\mathbf{r}\tau) \hat{\sigma}^a_{\alpha\alpha'} c_{\alpha'}(\mathbf{r}\tau)$ denotes the vector spin operator ($a = x, y, z$) with $\hat{\sigma}^a$ being the Pauli matrices. The unit vector $\mathbf{\Omega}(\mathbf{r}\tau) = [\sin\vartheta(\mathbf{r}\tau)\cos\varphi(\mathbf{r}\tau), \sin\vartheta(\mathbf{r}\tau)\sin\varphi(\mathbf{r}\tau), \cos\vartheta(\mathbf{r}\tau)]$ written in terms of polar angles labels varying in space-time spin quantization axis. The spin–rotation invariance is made explicit by performing the angular integration over $\mathbf{\Omega}(\mathbf{r}\tau)$ at each site and time. By decoupling spin and charge density terms in Eq.(3) using auxiliary fields $\varrho(\mathbf{r}\tau)$ and $iV(\mathbf{r}\tau)$ respectively, we write down the partition function in the form [9]

$$\mathcal{Z} \;=\; \int [\mathcal{D}\mathbf{\Omega}] \int [\mathcal{D}V \mathcal{D}\varrho] \int [\mathcal{D}\bar{c}\mathcal{D}c]\, e^{-\mathcal{S}[\mathbf{\Omega}, V, \varrho, \bar{c}, c]}. \tag{4}$$

where $[\mathcal{D}\mathbf{\Omega}] \equiv \prod_{\mathbf{r}\tau_k} \frac{\sin\vartheta(\mathbf{r}\tau_k) d\vartheta(\mathbf{r}\tau_k) d\varphi(\mathbf{r}\tau_k)}{4\pi}$ is the spin-angular integration measure. The effective action reads:

$$\begin{aligned} \mathcal{S}[\mathbf{\Omega}, V, \varrho, \bar{c}, c] \;&=\; \sum_{\mathbf{r}} \int_0^{\beta} d\tau \left[\frac{\varrho^2(\mathbf{r}\tau)}{U} + \frac{V^2(\mathbf{r}\tau)}{U} + iV(\mathbf{r}\tau) n(\mathbf{r}\tau) + 2\varrho(\mathbf{r}\tau) \mathbf{\Omega}(\mathbf{r}\tau) \cdot \mathbf{S}(\mathbf{r}\tau) \right] \\ &+\; \mathcal{S}_B[\bar{c}, c] + \int_0^{\beta} d\tau \mathcal{H}_t[\bar{c}, c]. \end{aligned} \tag{5}$$

Simple Hartree-Fock theory won't work for a Hubbard model in which U is the largest energy in the problem. One has to isolate strongly fluctuating modes generated by the Hubbard term according to the charge

U(1) and spin SU(2) symmetries. To this end we write the fluctuating "imaginary chemical potential" $iV(\mathbf{r}\tau)$ as a sum of a static $V_0(\mathbf{r})$ and periodic function $V(\mathbf{r}\tau) = V_0(\mathbf{r}) + \tilde{V}(\mathbf{r}\tau)$ using Fourier series $\tilde{V}(\mathbf{r}\tau) = \frac{1}{\beta}\sum_{n=1}^{\infty}[\tilde{V}(\mathbf{r}\omega_n)e^{i\omega_n\tau} + c.c.]$ with $\omega_n = 2\pi n/\beta$ ($n = 0, \pm 1, \pm 2$) being the (Bose) Matsubara frequencies. Now, we introduce the U(1) *phase* field $\phi(\mathbf{r}\tau)$ via the Faraday–type relation [8]

$$\dot{\phi}(\mathbf{r}\tau) \equiv \frac{\partial\phi(\mathbf{r}\tau)}{\partial\tau} = e^{-i\phi(\mathbf{r}\tau)}\frac{1}{i}\frac{\partial}{\partial\tau}e^{i\phi(\mathbf{r}\tau)} = \tilde{V}(\mathbf{r}\tau). \tag{6}$$

Furthermore, by performing the local gauge transformation to the *new* fermionic variables $f_\alpha(\mathbf{r}\tau)$:

$$\left[\begin{array}{c} c_\alpha(\mathbf{r}\tau) \\ \bar{c}_\alpha(\mathbf{r}\tau) \end{array}\right] = \left[\begin{array}{cc} z(\mathbf{r}\tau) & 0 \\ 0 & \bar{z}(\mathbf{r}\tau) \end{array}\right]\left[\begin{array}{c} f_\alpha(\mathbf{r}\tau) \\ \bar{f}_\alpha(\mathbf{r}\tau) \end{array}\right] \tag{7}$$

where the unimodular parameter $|z(\mathbf{r}\tau)|^2 = 1$ satisfies $z(\mathbf{r}\tau) = e^{i\phi(\mathbf{r}\tau)}$, we remove the imaginary term $i\int_0^\beta d\tau\tilde{V}(\mathbf{r}\tau)n(\mathbf{r}\tau)$ for all the Fourier modes of the $V(\mathbf{r}\tau)$ field, except for the zero frequency. Subsequent SU(2) transformation from $f_\alpha(\mathbf{r}\tau)$ to $h_\alpha(\mathbf{r}\tau)$ operators,

$$\left[\begin{array}{c} f_\uparrow(\mathbf{r}\tau) \\ f_\downarrow(\mathbf{r}\tau) \end{array}\right] = \left[\begin{array}{cc} \zeta_\uparrow(\mathbf{r}\tau) & -\bar{\zeta}_\downarrow(\mathbf{r}\tau) \\ \zeta_\downarrow(\mathbf{r}\tau) & \bar{\zeta}_\uparrow(\mathbf{r}\tau) \end{array}\right]\left[\begin{array}{c} h_1(\mathbf{r}\tau) \\ h_\downarrow(\mathbf{r}\tau) \end{array}\right], \quad \mathbf{R}(\mathbf{r}\tau) \equiv \left[\begin{array}{cc} \zeta_\uparrow(\mathbf{r}\tau) & -\bar{\zeta}_\downarrow(\mathbf{r}\tau) \\ \zeta_\downarrow(\mathbf{r}\tau) & \bar{\zeta}_\uparrow(\mathbf{r}\tau) \end{array}\right] \tag{8}$$

with the constraint $|\zeta_\uparrow(\mathbf{r}\tau)|^2 + |\zeta_\downarrow(\mathbf{r}\tau)|^2 = 1$ takes away the rotational dependence on $\mathbf{\Omega}(\mathbf{r}\tau)$ in the spin sector. This is done by means of the Hopf map $\mathbf{R}(\mathbf{r}\tau)\hat{\sigma}^z\mathbf{R}^\dagger(\mathbf{r}\tau) = \hat{\sigma}\cdot\mathbf{\Omega}(\mathbf{r}\tau)$ that is based on the enlargement from two-sphere S_2 to the three-sphere $S_3 \sim SU(2)$. The unimodular constraint can be resolved by using the parametrization

$$\begin{aligned} \zeta_\uparrow(\mathbf{r}\tau) &= e^{-\frac{i}{2}[\varphi(\mathbf{r}\tau)+\chi(\mathbf{r}\tau)]}\cos\left[\frac{\vartheta(\mathbf{r}\tau)}{2}\right] \\ \zeta_\downarrow(\mathbf{r}\tau) &= e^{\frac{i}{2}[\varphi(\mathbf{r}\tau)-\chi(\mathbf{r}\tau)]}\sin\left[\frac{\vartheta(\mathbf{r}\tau)}{2}\right] \end{aligned} \tag{9}$$

with the Euler angular variables $\varphi(\mathbf{r}\tau), \vartheta(\mathbf{r}\tau)$ and $\chi(\mathbf{r}\tau)$, respectively. Here, the extra variable $\chi(\mathbf{r}\tau)$ represents the U(1) gauge freedom of the theory as a consequence of $S_2 \to S_3$ mapping. One can summarize Eqs (7) and (8) by the single joint gauge transformation exhibiting electron operator factorization $c_\alpha(\mathbf{r}\tau) = \sum_{\alpha'}\mathcal{U}_{\alpha\alpha'}(\mathbf{r}\tau)h_{\alpha'}(\mathbf{r}\tau)$, where $\mathcal{U}(\mathbf{r}\tau) = z(\mathbf{r}\tau)\mathbf{R}(\mathbf{r}\tau)$ is a U(2) matrix which rotates the spin-quantization axis at site $\mathbf{r}$ and time τ. This reflects the composite nature of the interacting electron formed from bosonic spinorial and charge degrees of freedom given by $R_{\alpha\alpha'}(\mathbf{r}\tau)$ and $z(\mathbf{r}\tau)$, respectively as well as remaining fermionic part $h_\alpha(\mathbf{r}\tau)$. Accordingly, the integration measure over the group manifold becomes

$$\int[\mathcal{D}\phi\mathcal{D}\mathbf{\Omega}] \equiv \sum_{\{m(\mathbf{r})\}}\prod_{\mathbf{r}}\int_0^{2\pi}d\phi_0(\mathbf{r})\int d\mathbf{\Omega}_0(\mathbf{r})\int_{\phi(\mathbf{r}0)=\phi_0(\mathbf{r})}^{\phi(\mathbf{r}\beta)=\phi_0(\mathbf{r})+2\pi m(\mathbf{r})}\mathcal{D}\phi(\mathbf{r}\tau)\int_{\mathbf{\Omega}(\mathbf{r}0)=\mathbf{\Omega}_0}^{\mathbf{\Omega}(\mathbf{r}\beta)=\mathbf{\Omega}_0}\mathcal{D}\mathbf{\Omega}(\mathbf{r}\tau) \tag{10}$$

where $\int d\mathbf{\Omega}\cdots = \frac{1}{4\pi}\int_0^\pi\sin\theta d\vartheta\int_0^{2\pi}d\varphi\ldots$ and $[\mathcal{D}\mathbf{\Omega}(\mathbf{r}\tau)] = \prod_k d\mathbf{\Omega}(\mathbf{r}\tau_k)$. Here, $m \in Z$ labels equivalence classes of homotopically connected paths [10].

4 Effective phase-angular action

Anticipating that spatial and temporal fluctuations of the fields $V_0(\mathbf{r})$ and $\varrho(\mathbf{r}\tau)$ will be energetically penalised, since they are gaped and decouple from the angular and phase variables. Therefore, in order to make further progress towards we next subject the functional to a saddle point analysis. The expectation value of the static (zero frequency) part of the fluctuating electrochemical potential $V_0(r)$ we calculate by the saddle point method to give $V_0(r) = i\left(\mu - \frac{U}{2}n_h\right) \equiv i\bar{\mu}$ where $\bar{\mu}$ is the chemical potential with a

Hartree shift originating from the saddle-point value of the static variable $V_0(\mathbf{r})$ with $n_h = n_{h\uparrow} + n_{h\downarrow}$ and $n_{h\alpha} = \langle \bar{h}_\alpha(\mathbf{r}\tau) h_\alpha(\mathbf{r}\tau)\rangle$. Similarly in the magnetic sector

$$\rho(\mathbf{r}\tau) = \begin{cases} (-1)^{\mathbf{r}}\Delta_c \\ \pm\Delta_c \end{cases} \tag{11}$$

where $\Delta_c = U\langle S^z(\mathbf{r}\tau\rangle$ sets the magnitude for the Mott-charge gap. The two choices delineated in Eq.(11) correspond to the saddle point of the "antifferomagnetic" (with staggering Δ_c) or "ferromagnetic type". Note that the notion ferromagnetic (antifferomagnetic) here does not mean an actual long–range ordering - for this the angular spin-quantization variables have to be ordered as well. In the new variables the action in Eq.(5) assumes the form $\mathcal{S}\left[\mathbf{\Omega}, \phi, \varrho, \bar{h}, h\right] = \mathcal{S}_B[\bar{h}, h] + \int_0^\beta d\tau \mathcal{H}_{\mathbf{\Omega},\phi}[\rho, \bar{h}, h] + \mathcal{S}_0\left[\phi\right] + 2\sum_{\mathbf{r}}\int_0^\beta d\tau \mathbf{A}(\mathbf{r}\tau)\cdot \mathbf{S}_h(\mathbf{r}\tau)$, where $\mathbf{S}_h(\mathbf{r}\tau) = \frac{1}{2}\sum_{\alpha\gamma}\bar{h}_\alpha(\mathbf{r}\tau)\hat{\sigma}_{\alpha\gamma}h_\gamma(\mathbf{r}\tau)$. Furthermore,

$$S_0[\phi] = \sum_{\mathbf{r}}\int_0^\beta d\tau\left[\frac{\dot{\phi}^2(\mathbf{r}\tau)}{U} + \frac{1}{i}\frac{2\mu}{U}\dot{\phi}(\mathbf{r}\tau)\right] \tag{12}$$

stands for the kinetic and Berry term of the U(1) phase field in the charge sector.The SU(2) gauge transformation in Eq.(8) and the fermionic Berry term generate SU(2) potentials given by $\mathbf{R}^\dagger(\mathbf{r}\tau)\partial_\tau\mathbf{R}(\mathbf{r}\tau) = \mathbf{R}^\dagger\left(\dot{\varphi}\frac{\partial}{\partial\varphi} + \dot{\vartheta}\frac{\partial}{\partial\vartheta} + \dot{\chi}\frac{\partial}{\partial\chi}\right)\mathbf{R} = -\hat{\sigma}\cdot\mathbf{A}(\mathbf{r}\tau)$, where

$$\begin{aligned} A^x(\mathbf{r}\tau) &= \frac{i}{2}\dot{\vartheta}(\mathbf{r}\tau)\sin\chi(\mathbf{r}\tau) - \frac{i}{2}\dot{\varphi}(\mathbf{r}\tau)\sin\theta(\mathbf{r}\tau)\cos\chi(\mathbf{r}\tau) \\ A^y(\mathbf{r}\tau) &= \frac{i}{2}\dot{\vartheta}(\mathbf{r}\tau)\cos\chi(\mathbf{r}\tau) + \frac{i}{2}\dot{\varphi}(\mathbf{r}\tau)\sin\theta(\mathbf{r}\tau)\sin\chi(\mathbf{r}\tau) \\ A^z(\mathbf{r}\tau) &= \frac{i}{2}\dot{\varphi}(\mathbf{r}\tau)\cos\vartheta(\mathbf{r}\tau) + \frac{i}{2}\dot{\chi}(\mathbf{r}\tau). \end{aligned} \tag{13}$$

are the SU(2) gauge potentials. The fermionic sector, in turn, is governed by the effective Hamiltonian

$$\begin{aligned} &\mathcal{H}_{\mathbf{\Omega},\phi} = \sum_{\mathbf{r}}\varrho(\mathbf{r}\tau)[\bar{h}_\uparrow(\mathbf{r}\tau)h_\uparrow(\mathbf{r}\tau) - \bar{h}_\downarrow(\mathbf{r}\tau)h_\downarrow(\mathbf{r}\tau)] \\ &-t\sum_{\langle\mathbf{r},\mathbf{r}'\rangle}\sum_{\alpha\gamma}\left[\mathcal{U}^\dagger(\mathbf{r}\tau)\mathcal{U}(\mathbf{r}'\tau)\right]_{\alpha\gamma}\bar{h}_\alpha(\mathbf{r}\tau)h_\gamma(\mathbf{r}'\tau) - \bar{\mu}\sum_{\mathbf{r}\alpha}\bar{h}_\alpha(\mathbf{r}\tau)h_\alpha(\mathbf{r}\tau), \end{aligned} \tag{14}$$

The result of the gauge transformations is that we have managed to cast the strongly correlated problem into a system of mutually non-interacting fermions, submerged in the bath of strongly fluctuating U(1) and SU(2) fields whose dynamics is governed by the energy scale set by the Coulomb interaction U coupled to fermions via hopping term and with the Zeeman-type contribution with the massive field $\varrho(\mathbf{r}\tau)$.

In analogy to the charge U(1) field the SU(2) spin system exhibit emergent dynamics. Integration of fermions will generate the kinetic term for the SU(2) rotors

$$\begin{aligned} \mathcal{S}_0[\mathbf{\Omega}] &= -\frac{1}{2}\times 4\sum_{\mathbf{r}\mathbf{r}'}\int_0^\beta d\tau d\tau'\sum_{ab}A^a(\mathbf{r}\tau)A^b(\mathbf{r}'\tau') \\ &\times \sum_{\mathbf{r}'}\langle S_h^a(\mathbf{r}\tau)S_h^b(\mathbf{r}'\tau')\rangle - 2\sum_{\mathbf{r}\mathbf{r}'}\int_0^\beta d\tau\mathbf{A}(\mathbf{r}\tau)\cdot\langle\mathbf{S}_h(\mathbf{r}'\tau')\rangle \end{aligned} \tag{15}$$

with

$$\langle S_h^a(\mathbf{r}\tau)S_h^b(\mathbf{r}'\tau')\rangle = -\frac{1}{4}\times 2\delta_{ab}\frac{1}{\mathcal{E}_s}, \quad \langle S_h^z(\mathbf{r}\tau)\rangle = \frac{\Delta_c}{U} \tag{16}$$

In the AF phase, at the half-filling, it assumes the staggered form $\varrho(\mathbf{r}\tau) = \Delta_c(-1)^{\mathbf{r}}$ with Δ_c being the charge gap $\Delta_c \sim U/2$ for $U/t \gg 1$, with $E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_c^2}$, $\mathbf{Q} = (\pi,\pi)$ and $\varepsilon_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \bar{\mu}$. At zero

temperature $\lim_{T\to 0}\frac{1}{\mathcal{E}_s}=\frac{\Delta_c^2}{2E_{\mathbf{k}}^3}$ so that, in the limit $U/t\gg 1$ one obtains $\mathcal{E}_s\sim 2\Delta_c=U$. However, a nonzero value of Δ_c does not imply the existence of anfiferromagnetic long–range order. For this the angular degrees of freedom $\mathbf{\Omega}(\mathbf{r}\tau)$ have also to be ordered, whose low-lying excitations are in the form of spin waves. Therefore the kinetic term in the spin sector becomes $\mathcal{S}_0[\mathbf{\Omega}]=-\frac{1}{\mathcal{E}_s}\sum_{\mathbf{r}}\int_0^\beta d\tau\mathbf{A}(\mathbf{r}\tau)\cdot\mathbf{A}(\mathbf{r}\tau)$ so that the total kinetic energy $\mathcal{S}[\mathbf{\Omega},\phi]=\mathcal{S}_0[\phi]+\mathcal{S}_0[\mathbf{\Omega}]$ for U(1) and SU(2) rotors becomes

$$\begin{aligned}\mathcal{S}_0[\mathbf{\Omega},\phi] &= \sum_{\mathbf{r}}\int_0^\beta d\tau\left\{\frac{\dot\phi^2(\mathbf{r}\tau)}{U}+\frac{1}{i}\frac{2\mu}{U}\dot\phi(\mathbf{r}\tau)+\frac{1}{4\mathcal{E}_s}\left[\dot\vartheta^2(\mathbf{r}\tau)+\dot\varphi^2(\mathbf{r}\tau)+\dot\chi^2(\mathbf{r}\tau)\right.\right.\\ &+ \left.\left.2\dot\varphi(\mathbf{r}\tau)\dot\chi(\mathbf{r}\tau)\cos\vartheta(\mathbf{r}\tau)\right]+\frac{\Delta_c}{iU}\left[\dot\varphi(\mathbf{r}\tau)\cos\vartheta(\mathbf{r}\tau)+\dot\chi(\mathbf{r}\tau)\right]\right\}. \qquad (17)\end{aligned}$$

The distinctive feature of Eq.(17) is th presence of the geometric Berry contributions that signify topological features of the underlying field theory [11].

5 Summary

In this work we have observed that the important symmetries of the Hubbard model given by by the charge U(1) gauge and spin rotational SU(2) groups imply that the quantum dynamics is governed non-trivially by the multiply connected U(2)=U(1)$\otimes$SU(2) group manifold. Following this route we have developed a spin-charge unifying description for interacting electrons given by the Hubbard model. It is based on the time dependent local gauge transformations to disentangle the Coulomb interaction. The collective variables for charge and spin are isolated in a form of the space-time fluctuating phase field and the rotating spin quantization axis. As a result interacting electrons appear as a composite objects consisting of bare fermions with attached gauge fields. Our results form a general framework tailored to describe eg. both magnetic and superconducting order which is rooted in microscopic of strongly correlated electrons and their basic symmetries and interactions [12]. In particular, it will enable to identify the degrees of freedom of the hierarchy of low-energy effective theories, and the way they are associated with the global symmetries.

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